Structured Labels and Directed Paths

K. K. Nambiar
School of Computer and Systems Sciences
Jawaharlal Nehru University, New Delhi 110067, India

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Abstract—Directed paths are obtained by inverting a matrix with elements from a field, showing that it is unnecessary to go through the cumbersome procedure of inverting a matrix with elements from a division ring.

Keywords—Structured labels; Graphs; Directed paths.

1. INTRODUCTION

It has been shown in an earlier paper [1] that for the graph shown in Fig. 1, the entire set of directed paths can be represented by the matrix

$$\mathbf{A}^* = \begin{bmatrix} a^* + a^*b(ca^*b)^*ca^* & a^*b(ca^*b)^* \\ (ca^*b)^*ca^* & (ca^*b)^* \end{bmatrix}$$

where $x^* = (1-x)^{-1} = 1 + x + x^2 + \cdots$ and the elements of the matrix are assumed to be from a *division ring*. The purpose of this paper is to show that the same can be achieved in a simpler way if we use *structured labels* taken from a *field*.

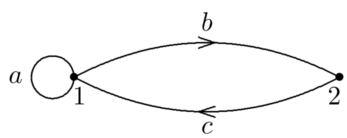

Fig. 1

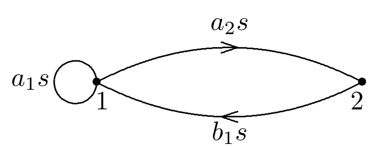

Fig. 2

2. DIRECTED PATHS

An example of the use of structured labels is shown in Fig. 2. Every node is represented by both a literal and and a number, i.e., nodes $1, 2, 3, \cdots$ are also designated as $a, b, c, \cdots$ respectively. Every edge is labelled by a subscripted literal: if an edge goes from node b to node 1, as in Fig. 2, that edge is given the label b_1s. All the edges are labelled in a similar fashion and these labels are taken to be elements from a field. With these notations, the adjacency matrix [2] of the graph in Fig. 2 can be written as

$$\mathbf{A} = \begin{bmatrix} a_1s & a_2s \\ b_1s & 0 \end{bmatrix}$$

The paths in the graph can now be obtained by inverting $(\mathbf{I} - \mathbf{A})$, where $\mathbf{I}$ is the unit matrix.

$$(\mathbf{I} - \mathbf{A})^{-1} = \begin{bmatrix} 1 & a_2s \\ b_1s & 1 - a_1s \end{bmatrix} \frac{1}{(1 - a_1s - a_2b_1s^2)}$$

The element (1,2) here gives the *pathset* from node 1 to node 2.

$$\begin{aligned} p_{12}(s) &= \frac{a_2 s}{1 - a_1 s - a_2 b_1 s^2} \\ &= a_2 s[1 + (a_1 s + a_2 b_1 s^2) + (a_1 s + a_2 b_1 s^2)^2 + \cdots] \\ &= a_2 s + a_1 a_2 s^2 + (a_1^2 a_2 + a_2^2 b_1) s^3 + (a_1^3 a_2 + 2 a_1 a_2^2 b_1) s^4 + \cdots \end{aligned}$$

The coefficient 2 appearing along with the string $a_1 a_2^2 b_1$ shows that the string can be unfolded as a path in two different ways, namely, $a_1 a_2 b_1 a_2$ and $a_2 b_1 a_1 a_2$. Written explicitly as paths

$$p_{12}(s) = a_2 s + a_1 a_2 s^2 + (a_1 a_1 a_2 + a_2 b_1 a_2) s^3 + (a_1 a_1 a_1 a_2 + a_1 a_2 b_1 a_2 + a_2 b_1 a_1 a_2) s^4 + \cdots$$

It should clear from the discussion above that $p_{12}(s)$ specifies the entire set of paths from node 1 to node 2 in the graph.

3. CONCLUSION

If we invert $(\mathbf{I} - \mathbf{A})$ as a matrix with elements from a division ring, we get the *explicit form* of the pathset we are considering. Then

$$p_{12}(s) = (a_1 s)^* (a_2 s)(b_1 s (a_1 s)^* a_2 s)^*$$

It is easy to see that we can *walkthrough* the pathset if we read the explicit form from left to right. Note that the explicit form reduces to the *implicit form*

$$\frac{a_2 s}{1 - a_1 s - a_2 b_1 s^2}$$

when we substitute $(1 - x)^{-1}$ for x^*. An interesting aside here is that $p_{12}(s)$ becomes the generating function of the Fibonacci sequence when we put $a_1 = a_2 = b_1 = 1$.

REFERENCES

1. K. K. Nambiar, Matrices with Elements from a Division Ring, *Mathematical and Computer Modelling,* to be published.
2. N. Deo, *Graph Theory,* Prentice Hall of India, New Delhi, India, (1984).