A NOTE ON INDUCTIVE PROBABILITY

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(Received August 1993)

Abstract—Probabilistic induction allows us to conclude that the probability of a hypothesis being true increases when evidences in favor of the hypothesis occur. A claim made by Karl Popper and David Miller that probabilistic induction is impossible is shown to be untenable, using an example.

Keywords: Probabilistic induction; Evidence; Credibility; Confirmation.

1. INTRODUCTION

The purpose of this paper is to clarify certain issues raised by Karl Popper and David Miller on inductive probability [1] and controvert their assertion about the "the impossibility of inductive probability." In the sequel it is shown that probabilistic induction is possible, at least in some special cases. See [2] for further reservations regarding the Popper-Miller assertion. We make our notations and definitions clear before we proceed further. As far as possible, the notations in [1] have been used.

A proposition h whose truth we want to investigate, we call a *hypothesis*. A proposition e such that $h \Rightarrow e$ is true, we call an *evidence* of h. If the evidence e is such that it is also true that $e \Rightarrow h$, then we call it a *confirmation* of h. Assuming that we have a probabilistic system, we can investigate the probability of the proposition $e \Rightarrow h$ being true, where e is an evidence of h. We call this probability $p(e \Rightarrow h)$, the *credibility* of the evidence e. We want to show that the credibility increases when the evidences pile up. In the following we assume S_1 to be the initial system and every time an evidence occurs we consider the resulting system as a new system. Further we will assume that at no time a counterevidence turns up. Thus we have evidences turning up continuously and the series of systems designated as S_n. A fact we can reasonably assume is that if the hypothesis is not true, the probability of an evidence turning up remains stationary and does not depend on the previous evidences. Thus,

$$p(e_n|\overline{h}_n) = p(e_{n+1}|\overline{h}_{n+1}). \tag{1}$$

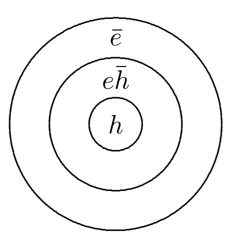

Fig. 1. System S

To firm up the terminology, consider the example of a freak coin with heads on both sides. The problem is to decide whether a given coin is freak or not by tossing it and observing the outcome. Here the hypothesis h is that *the coin is freak*. The evidence e is that *a head has turned up*. It is quite obvious that the probability of the coin being freak increases continuously if the heads are turning up every time.

In the Venn diagram shown in Fig. 1, the innermost circle represents the event h, the outermost circle represents the universal event and the middle circle represents the event e. The outer annular region represents the event $\overline{e}$ and the inner annular region $e\overline{h}$, as marked. We will attach subscripts to these symbols when they refer to a particular system. Thus for the system S_n we write $p(h_n) = a_n$, $p(e_n\overline{h}_n) = b_n$, $p(\overline{e}_n) = c_n$.

2. PROBABILISTIC INDUCTION

THEOREM. a_n monotonically increases, while b_n and c_n monotonically decrease with n. Further,

$$\lim_{n\to\infty} a_n = 1$$

$$\lim_{n\to\infty} b_n = 0$$

$$\lim_{n\to\infty} c_n = 0.$$

Also,

$$\lim_{n\to\infty} p(e_n \Rightarrow h_n) = 1$$

i.e., the credibility of the evidence increases when evidences turn up and finally the evidence becomes a confirmation.

PROOF. To avoid pathological conditions, we assume that a_1, b_1, and c_1 are positive. Since $p(h_{n+1})$ is by definition equal to $p(h_n|e_n)$, we can use Fig. 1 to write,

$$a_{n+1} = p(h_{n+1}) = p(h_n|e_n) = \frac{p(h_n e_n)}{p(e_n)} = \frac{p(h_n)}{p(e_n)} = \frac{a_n}{a_n + b_n}. \tag{2}$$

Using (1),

$$b_{n+1} = p(e_{n+1}\overline{h}_{n+1}) = p(e_{n+1}|\overline{h}_{n+1})p(\overline{h}_{n+1}) = p(e_n|\overline{h}_n)p(\overline{h}_{n+1}) = \frac{b_n}{(b_n + c_n)}\frac{b_n}{(a_n + b_n)} \tag{3}$$

and

$$c_{n+1} = \frac{b_n}{(a_n + b_n)}\frac{c_n}{(b_n + c_n)}. \tag{4}$$

From (3) and (4),

$$\frac{b_n}{c_n} = \frac{b_1}{c_1}. \tag{5}$$

Using (2) to (5),

$$b_{n+1} = (1 - a_{n+1})\frac{b_n}{b_n + c_n} = (1 - a_{n+1})\frac{b_1}{b_1 + c_1} \tag{6}$$

$$c_{n+1} = (1 - a_{n+1})\frac{c_n}{b_n + c_n} = (1 - a_{n+1})\frac{c_1}{b_1 + c_1}. \tag{7}$$

These equations show that a_n monotonically increases, and b_n and c_n monotonically decrease. Since a_n cannot be greater than one, it must have a limit as n tends to infinity. In other words,

$$\lim_{n\to\infty} \frac{a_n}{a_{n+1}} = 1.$$

Using (2),

$$\lim_{n\to\infty} (a_n + b_n) = \lim_{n\to\infty} \frac{a_n}{a_{n+1}} = 1.$$

Since $a_n + b_n + c_n = 1$,

$$\lim_{n\to\infty} c_n = 0. \tag{8}$$

Using (5) and (8),

$$\lim_{n\to\infty} b_n = \frac{b_1}{c_1} \lim_{n\to\infty} c_n = 0. \tag{9}$$

Since both b_n and c_n tend to zero,

$$\lim_{n\to\infty} a_n = 1.$$

Since b_n decreases monotonically and $(e_n \Rightarrow h_n) = (\bar{e}_n \vee h_n)$, by definition, it is clear that $p(e_n \Rightarrow h_n) = p(\bar{e}_n \vee h_n) = 1 - b_n$ increases monotonically. From (9) it immediately follows that

$$\lim_{n\to\infty} p(e_n \Rightarrow h_n) = 1.$$

Note that we have used only the axioms of elementary probability theory and this can be easily verified by working out in detail the example of the freak coin mentioned earlier. Finally, it should be stated that the following fact is of crucial significance for removing the confusion in Popper-Miller argument [1]: While it is true that $p([e_1 \Rightarrow h_1]|e_1) = p(h_1|e_1) = p(h_2) = a_2$, we must recognise that $p([e_1 \Rightarrow h_1]|e_1)$ is not the same as $p([e_2 \Rightarrow h_2]|e_1) = p(e_2 \Rightarrow h_2) = a_2 + c_2$. Further, it is easy to see from (2) that in the pathological case when $a_1 = 0$, a_n does not increase monotonically with n. This may also be another possible reason for the confusion in [1].

References

1. Popper K. and Miller D., *Nature* (Letters), **30**, 687–688, (1983).
2. Good, I. J. , A Suspicious Feature of the Popper-Miller Argument, *Philosophy of Science*, **57**, 533–536, (1990).