

# Generalized Continuum Hypothesis and the Axiom of Combinatorial Sets

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## Prologue

In an earlier paper [1], intuitive set theory (IST) was defined as the theory we get when we add the two axioms, *monotonicity* and *fusion*, to ZF theory. Here we attempt to replace the axiom of monotonicity with a simpler axiom we call, *axiom of combinatorial sets*.

## Axiom of Combinatorial Sets

If  $k$  is an ordinal, we write  $\binom{\aleph_\alpha}{k}$  for the cardinality of the set of all subsets of  $\aleph_\alpha$  with cardinality of  $k$ .

**Axiom of Combinatorial Sets:**

$$\aleph_{\alpha+1} = \binom{\aleph_\alpha}{\aleph_\alpha}.$$

## Derivation

We derive the Generalized Continuum Hypothesis from the axiom of combinatorial sets as below:

$$2^{\aleph_\alpha} = \binom{\aleph_\alpha}{0} + \binom{\aleph_\alpha}{1} + \binom{\aleph_\alpha}{2} + \cdots + \binom{\aleph_\alpha}{\aleph_0} + \cdots + \binom{\aleph_\alpha}{\aleph_\alpha}.$$

Note that  $\binom{\aleph_\alpha}{1} = \aleph_\alpha$ . Since, there are  $\aleph_\alpha$  terms in this addition and  $\binom{\aleph_\alpha}{k}$  is a monotonically nondecreasing function of  $k$ , we can conclude that

$$2^{\aleph_\alpha} = \binom{\aleph_\alpha}{\aleph_\alpha}.$$

Using axiom of combinatorial sets, we get

$$2^{\aleph_\alpha} = \aleph_{\alpha+1}.$$

## Epilogue

In view of the fact that we can derive the generalized continuum hypothesis from the axiom of combinatorial sets, we can replace the axiom of monotonicity [1, 2] with the axiom of combinatorial sets, in the definition of intuitive set theory.

## References

- [1] K. K. Nambiar, *Intuitive Set Theory*, Computers and Mathematics with Applications **39** (1999), no. 1-2, 183–185.
- [2] K. K. Nambiar, *Real Set Theory*, Computers and Mathematics with Applications **38** (1999), no. 7-8, 167–171.