UNCERTAINTY PRINCIPLE OF PHANTOM MECHANICS

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Abstract. A mathematical point traveling at random inside a unit interval according to a probability density function is called a phantom and its motion studied.

Keywords—phantom mechanics; localizing slit; constraining effort; restraining calculation.

1. Introduction

The purpose of this paper is to show that not only the particles of physics, but even a mathematical point in motion is governed by the uncertainty principle.

2. Walsh Functions

The first four of the well-known Walsh functions [1] are shown in Figure 1. They are orthonormal functions defined in the interval $(0, 1)$, and take values only $+1$ and -1. An interesting property of these functions is that the product of any two Walsh functions is another Walsh function.

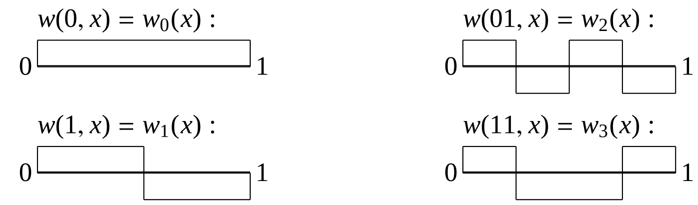

Figure 1. Walsh Functions

Consider the probability density function $p(x)$ shown in Figure 2.

Figure 2. Functions $p(x)$ and $\lg p(x)$

Since the functions in Figure 1 are orthonormal, we can write $p(x)$ and $\lg p(x)$ in terms of those functions. Omitting the routine calculations, we have

$$\begin{aligned} p(x) &= w_0(x) - 0.25w_1(x) + 0.25w_2(x) - 0.50w_3(x), \\ \lg p(x) &= -0.25w_0(x) - 0.25w_1(x) + 0.25w_2(x) - 0.75w_3(x). \end{aligned}$$

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3. Uncertainty Principle

We use Shannon's entropy [3] for defining a measure of the spread of a probability density function. The *localizing slit* of the phantom is defined as

$$S = 2^H,$$

where

$$H = -\int_0^1 p(x) \lg p(x)\, dx.$$

We define *restraining calculation* C on the phantom as the sum of the products of the corresponding coefficients in the Walsh series expansion of $p(x)$ and $\lg p(x)$, and the *constraining effort* on the phantom as

$$E = 2^C.$$

For our example, it easy to see that, $S = 2^{-0.25}$ and $E = 2^{0.25}$, and hence, $SE = 1$.

We will take up another example to justify the terminology used here. Note that the binary sequence

$$0.10 \star \star \star \star \ldots$$

can be used to represent the interval $[0.50, 0.75)$, if we assume that each $\star$ can be replaced by either a 0 or 1. If we consider a phantom travelling with uniform probability within this interval of length 0.25, we can write $p(x)$ and $\lg p(x)$ as

$$\begin{aligned} p(x) &= w_0(x) - w_1(x) + w_2(x) - w_3(x), \\ \lg p(x) &= 0.5w_0(x) - 0.5w_1(x) + 0.5w_2(x) - 0.5w_3(x). \end{aligned}$$

From this, we get the restraining calculation as

$$C = 2,$$

which is what it should be, since we had to make two bits of calculation to drive the phantom into the interval $[0.50, 0.75)$.

Again, for this example, we have $S = 2^{-2}$ and $E = 2^2$, and hence, $SE = 1$. We have used the first four Walsh functions for explaining the examples, only for clarity of exposition. We can consider the entire set of Walsh functions and write the uncertainty principle of phantom mechanics for an arbitrary probablity density function:

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Uncertainty Principle

***Slit* × *Effort* = 1**

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Note that we have used only the elementary concepts of linear vector spaces [2] in arriving at our results.

4. Conclusion

If we scrutinize the analysis here, it will be seen that our arguments will go through, even if we replace the phantom by an ordinary particle of physics. Hence, we may call $SE = 1$ as the *Refined Uncertainty Principle* of quantum mechanics.

References

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